

RST Engineering

13993 Downwind Court Grass Valley CA 95945 530.272.2203 voice only www.rstengineering.com

As pilots concerned with all aspects of our environment, we have noticed that the sun does not shine on our faces the same all year round. As a matter of fact, you can only get sun vertically on your face between two lines of latitude (and even then only two days a year). These are the Tropic of Cancer ( $23.5^{\circ}$ north of the equator) and the Tropic of Capricorn ( $23.5^{\circ}$ south of the equator).

And please at this point of time, let me make a general statement. My answers will NOT be accurate to half a degree. One degree accuracy is what you can expect from me. I will calculate these values above (for example) as $24^{\circ}$ in all the rest of my equations

In North America, this means that if we want maximum solar radiation (power) we will have to point our faces (and our solar generating array) south all year long. And not just ANY south, but TRUE south. The sun couldn't care less where that huge magnet in Greenland is; instead it cares where the earth is actually rotating on its axis. The true north-south pole line. (And please note that I use the word "solar array" equivalent to "solar cell")


I would like to pause right here and acknowledge that I will be disinheriting roughly half of the world with my analyses and equations. I am going to do equations that are valid for north of the equator. For my colleagues in Australia, South America, half of Africa, and the South Indian Ocean countries, the equations will be incorrect, but inversely incorrect. Anybody in this part of the world that wants to correct my equations for south of the equator, please be my guest and be assured that I do NOT have these equations copyrighted. Feel free to correct them for your part of the world and share.

Having said that, I think it is obvious that any solar power project is going to be impacted by (a) their latitude and (b) the time of year that they are trying to pry power from the sun. For example, I'm sitting here in Northern California, about 500 miles north of Los Angeles and San Diego. My brother in San Diego is going to have to pitch his solar cells up much more that I to get maximum energy, and my colleagues in Oshkosh WI are going to have to pitch their solar array much further down than I am for optimum results. Not only that, but my brother Don in San Diego is going to get more energy over the year than I am, but Dick Knapinski (Director of Communications for EAA-OSH) is going to get less energy that I will. Fact of life and nothing any of us can do except move.

Let's do the first equation (all these equations will be on a series of spreadsheets, so you don't really have to copy them down).

The question is, "Where do I point my solar array for best efficiency?"
The first answer we must have is, "What is your magnetic declination/variation?" which is a fancy way of asking "How much east or west is your hangar is the line from the Greenland earth magnet to the actual True North pole. Most of us don't have a "True North" compass (with the exception of a few of our colleagues with this built into our cell phones) so the equation is:

> True North = Compass Reading minus Declination (for easterly declination) True North = Compass Reading plus Declination (for westerly declination)

So, here I sit at Grass Valley Airport $13^{\circ}$ easterly declination trying to figure out where I point my solar array. True North is to the LEFT of Greenland (facing north), so it is correct that if I point my compass $13^{\circ} \mathrm{CCW}$ ("minus Declination") to $347^{\circ}$ then the compass should be pointed to True North. And so it is. I point my solar array DUE TRUE SOUTH, which is $180^{\circ}$ from true north, or $167^{\circ}$ on the compass

So where do we get this "declination (variation)" information? http://www.gcmap.com/airport/KGOO has this information for any airport in the United States:

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Type: Airport (Airfield)
Use: Public/Civil
Latitude: 39`13'27"N (39.224056)
Longitude:12100'09"W (-121.002555)
Datum: WGS 1984
Elevation: 3157 ft (962 m)
Variation:13.44*E (WMM2020 magnetic declination)
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This is enough information for you to point your solar array in the right direction. Now we have to figure out what the correct 'pitch" angle is for you and your time of year. On to the next subject "Pitch Angle".

## PITCH (TILT) ANGLE

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We have all seen the sun dip down closer to the southern horizon in winter and rebound halfway up in spring, and fully up (although still not overhead in the USA lower 48) as far as it is going to go in summer. If you take a look at "Solar Primer" on this web page you will see the lines Tropic of Cancer and Tropic of Capricorn. The sun is directly over Cancer on the first day of summer (summer solstice) and then winds itself down to Capricorn on the first day of winter (winter solstice). It is directly over the equator on both the first day of Spring and the first day of Fall (vernal and autumnal equinoxes respectively). If you plot sun position between these two limits you will find that the sun actually traces out a pretty accurate "sine wave" over the course of a year.

For a solar panel to be most efficient it has to be pointed directly at the sun at all times. Unfortunately for us, we can only make a reasonable compromise at this pointing without a lot of motors and mickey-motion to move the solar panels to point directly at the sun during daylight hours. And, for small systems like ours, those motors and motion take nearly as much power to operate as they gain in power output. But this sort of system is a benchmark that we can measure our compromise against. There is a name for this, and it is "solar insolance". If we define the system where we perfectly track the sun with machinery, and call it $100 \%$, then we can measure our compromise against it as a loss of perfection in percentage.

Here is the deal. The perfectionist can climb up onto the roof and adjust the angle of the solar panel once a day in terms of small fractions of a degree each day. In construction, we call this building a house with a micrometer and the builder is called an anal fool. Or, we can adjust the angle once and leave it there permanently. In construction we call this "measure it with a boot, mark it with a crayon, and cut it with an axe". Somewhere between these two is a reasonable compromise, and you get to define "reasonable:" for your own installation.

Here is an image of what we call a "pitch (tilt) angle." In winter the angle is nearly $70^{\circ}$ and in summer a bit less than $20^{\circ}$ in these images. Halfway in between ( $40^{\circ}$ or so) for Spring and Fall.

## Seasonal Solar Array Tilt Angles



Winter
Solstice


Some of the literature just says, "Make that angle between the horizontal roof and the solar array equal to your latitude plus a diddle constant" and never change it. The math says that you have dropped to nearly $70 \%$ of perfect.

Some of the literature says, "Change that angle in spring and fall to the best angle." Now you’ve hopped up to nearly $75 \%$ of perfect.

Some of the literature says, "Change that angle four times a year." You've gone up to $76 \%$ of perfect. Is that last $1 \%$ really worth climbing up on the roof in the snow and the beating sun? That's for YOU to decide.

Some of the literature gives equations for every day of the year. My momma didn't raise no fool. Do that sort of picking flyspecs out of the pepper if you wish.

Here are the equations. I'm going to use L as your latitude.
My latitude here at Grass Valley Intentional Airpatch is $39.2240556^{\circ}$. Again, taking things out beyond zero decimal places is foolishness. L (Grass Valley) $=39^{\circ}$ for the examples.

Single best (never change it) angle $=(0.75 * \mathrm{~L})+3^{\circ}$.
Example: $(0.75 * 39)+3=32^{\circ}$ Cant that sucker up 32 degrees from rooftop flat and weld it into place. Efficiency? About 70\% of perfect.

Change it around April $1^{\text {st }}$ and September $1^{\text {st }}$ every year?
Summer Angle $=(0.90 * L)-20^{\circ}$. Winter angle $=(0.90 * L)+20^{\circ}$.
Example $(0.90 * 39)-20^{\circ}=15^{\circ}$ in the summer and $(0.90 * 39)+20=55^{\circ}$ in the winter.
Efficiency about 75\%.

Change it around March $15^{\text {th }}$ and August $15^{\text {th }}$ for Spring and Fall respectively, Summer on April $15^{\text {th }}$ and Winter on October $15^{\text {th }}$.
For Spring and Fall, angle $=\mathrm{L}-2^{\circ}\left(\right.$ Example $39-2=37^{\circ}$
For Summer, angle $=\left(0.9 *\right.$ L) -24 (Example $(0.9 * 39)-24=11^{\circ}$
For Winter, angle $=\left(0.9 *\right.$ L) +24 (Example $(0.9 * 39)+24=59^{\circ}$
Efficiency about 76\%
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It has come down to calculating how to make a mount so that we can set that pitch angle correctly. Depending on your point of view, the calculations are quite easy, confusing, or horribly complex. I'll try and do some easing you into the process.

First of all, we have a typical solar panel with a typical mount made of angle metal. Aluminum or steel are the preferred materials, but if you have a reason to use wood or some other material the calculations are identical.


Here is a representative setup on a 100 watt solar panel (back view). The black box to the far left is the electrical connection to the solar panel cells and play a very minor role in the calculations. The solar mount consists of two "triangles" of angle metal, one attached to the left side of the panel and one to the right side.

There is one long leg bolted directly to the solar panel itself. Now I'm not a Luddite, and I really admire the metric system of measurement. If I didn't have a toolbox with a few thousand dollars worth of "Imperial" ("English") tools I'd certainly love to be working in the millimeter-kilogram system. Unfortunately, almost all American-built aircraft use the Imperial standard. This panel and mount came with metric hardware and my first job was to go to the hardware store to get "American-English" hardware.

Not only that, but the hardware was stainless steel. There is a caveat in the installation manual that cautions that stainless has a nasty tendency to gall and freeze over time. According to the manual, you are supposed to use antiseize on the stainless threads, but those of us who have been using antiseize (both the graphite and copperloaded versions) know how messy that stuff is on spark plug threads inside the engine head. You can only imagine how messy it is on threads exposed to the environment. I used plain old galvanized hardware with galvanized nylon insert locknuts and galvanized washers. I will probably redo the hardware on 10 year intervals as the galvanize slowly wears off.

So, you have one 28 " long leg made of 1 " x 2 " x 0.12 " aluminum angle bracket material that fastens onto the solar panel with hex head galvanized steel bolts, flatwashers, and galvanized steel nylon insert locknuts. There are holes on both sides of this bracket, one for attachment to the solar panel and one for attachment to the "brace" (see below). These holes are spaced on 1.5" centers. We will call this leg "c"

Then, fastened to the bottom end (and it is arbitrary which end you call "bottom") is a 22 " long pivot arm reasonably allowed (but snug) to rotate from flat against the solar panel ( $0^{\circ}$ pitch angle) to vertical ( $90^{\circ}$ pitch angle). Holes as in the long leg above and on the same centers. One of these hole patterns is going to be used to fasten your solar panel to your roof and the other one will be attached to the brace. This is leg "b".

One more piece and our triangle is complete. There is an 11 " brace made from $3 / 4$ " x $11 / 4$ " angle aluminum with two holes on $103 / 4$ " spacing. This brace will be vital to our calculation of pitch angle. I'm not a mechanical engineer, just a poor electronic hack. Some ME is going to have to explain to me why I wouldn't use the same heavy angle material on the brace as on the other two arms. Seems to me the brace is taking the brunt of the stress. This is leg "a"

Schematically, this is what our triangle looks like:


Now, on to the mathematics to make that Tilt Angle "A".
From a math point of view, this is what our triangle looks like:


We have the side that is connected to the roof, and we call it "c" where c is measured in inches (or centimeters, or cubits, or anything you want, just use the same units for all three sides). We have side "b" that is bolted to the solar panel, and we have little side "a", the brace. What we really want to calculate is angle A, the pitch angle. We can also calculate angles $B$ and $C$ without too much extra math and you might just want that information for your records.

You may be a math wiz and the term arc-cosine does not ruffle your feathers. You may have had trouble with freshman algebra and never wanted to see another equation in your life. We can accommodate all in this explanation.

One perfectly understandable definition: arc-cosine, arc-cos, and $\cos ^{-1}$ on your scientific calculator mean exactly the same thing. If you can't find $\cos ^{-1}$ on your calculator, most any competent current $8^{\text {th }}$ grader can find it for you. Don't worry about WHAT it does, just accept it as a modern miracle of electronics.

We have three measurements that we've taken. The lengths of $a$ and $b$ and $c$. Here is how to calculate A (Pitch Angle).
$A($ in degrees $)=\cos ^{-1}\left(\left(b^{2}+c^{2}-a^{2}\right) /(2 * b * c)\right)$
Let's do an example to see how this works.
We number the holes that we can use on arms "c" and "b" from 0 (zero) to the maximum available on that arm. Let's say just for information that we use hole 9 on arm "c" and 7 on arm "b". With a distance of 1.5 " between each hole center-to-center.

Length "a" = 10.75" (a constant from the two holes on the brace)

Length "b" $=7$ * $1.5=10.5$ "
Length " $c$ " $=9$ * $1.5=13.5$ "
$\mathrm{a}^{2}=10.75^{2}=115.6$ ( a constant in all these calculations)
$b^{2}=10.5^{2}=110.3$
$c^{2}=13.5^{2}=182.3$
So, $\left(b^{2}+c^{2}-a^{2}\right)=177.0$
And $(2 * b * c)=283.5$
So $\left(\left(b^{2}+c^{2}-a^{2}\right) /(2 * b * c)\right)=0.624$
And using the handy calculator, $\cos ^{-1}(0.624)=51^{\circ}$
So, using the "c9" hole and the "b7" hole as described above, the pitch angle is $51^{\circ}$
As best I can determine using lab instruments accurate to a tenth of a degree, this is exactly the pitch angle on my solar panel.

Of course, these calculations just took me half an hour to be sure I didn't make a math or a calculator entry error. You only have to do it ONCE for your array, your latitude, and your magnetic deviation. But once is once too often when we now have wonderful things like spreadsheet calculators. That allows us to do "what if" I change holes, does that make it better. That spreadsheet is on this website.

Let me make a couple of observations.

1. For high pitch angles (winter conditions with high winds) it is mechanically more robust to use "b" and "c" holes to be about the same on each arm, plus or minus two holes from each other.
2. You may wish to reconsider again what the brace material is. Again, I'm not an ME, but it seems to me that keeping the "a" brace angle nearly $90^{\circ}$ between the b and c arms is best for the brace.


This image shows the wasted hardware that comes with this particular model of solar panel mount. I would really like to encourage providers of mounts to do three things:

1. Use Imperial-American hardware sizes;
2. Use plain old galvanized steel hardware instead of stainless;
3. Forget the fancy hand-wheel bolts ...

.... and use plain old hex bolts you can turn with a wrench.


And finally, one last reminder:


